$$\frac{A(x^2)-xB(x^2)}{A(x^2)+xB(x^2)}=r^{2x}; \text{ whence } \frac{A(x^2)}{x\,B(x^2)}=\frac{1+r^{2x}}{1-r^{2x}}=-\frac{r^x+r^{-x}}{r^x-r^{-x}};$$

$$\text{or, } \frac{A(x^2)}{B(x^2)}=-\frac{r^x+r^{-x}}{(1/x)\ (r^x-r^{-x})}.$$

$\therefore A(x^2)=(r^x+r^{-x}).\phi(x^2)$, $B(x^2)=-\frac{1}{x}(r^x-r^{-x}).\phi(x^2)$, where $\phi(x^2)$ is any function of x^2.

$\therefore f(x)=\frac{2}{r^x}.\phi(x^2)$.

II. **Solution by S. LEFSCHETZ, Ph. D., The University of Nebraska.**

The given functional equation can be written:

$$f(-x)r^{-x}=f(x)r^x.$$

Hence $f(x)r^x=\phi(x)$, where ϕ is any even function of x.

$\therefore f(x)=\frac{\phi(x)}{r^x}$.

Ex. $\phi(x)=\sum_0^\infty mA_m\cos mx$; $\phi(x)=\psi(x^2)$, etc.

Also solved by A. H. Holmes and J. Scheffer.

369. Proposed by WILLIAM HOOVER, Ph. D., Athens, Ohio.

If $f(m)=(1+x)^m$, and $f(n)=(1+x)^n$, why not obviously $f(m).f(n)=(1+x)^{m+n}=f(m+n)$?

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

The proof given of $f(m)f(n)=f(m+n)$ is certainly incontestable; but it may also be proved directly thus: Differentiating with reference to m and n separately, we have

$$f(n)\,f'(m)=f'(m+n),\quad f'(n)\,f(m)=f'(m+n).$$

$\therefore f(n)\ f'(m)=f'(n)\ f(m)$; $\therefore \frac{f'(m)}{f(m)}=$a constant$=a$ (say).

$\therefore f(m)=a^m$.

Solutions of 367 were received from A. M. Harding, Elmer Schuyler, S. Lefschetz, and the Proposer.